CASE 11

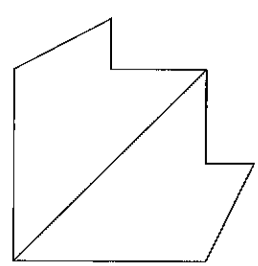
Crazy cakes

Phoebe

GRADE 4, FEBRUARY

My fourth-grade students just started a fractions unit that uses an area model of fractions. In the introductory activity, students divide paper "crazy cakes" (irregular polygonal shapes) for two people to share equally.* Working with partners, they had to divide each cake and be able to prove that the resulting pieces were halves.

After students had finished dividing a number of cakes, we began a discussion. I picked what I expected was a fairly easy cake to divide and put it on the overhead. Elsa drew a line to show how she split it. I then turned the question to the class, "Can you prove or disprove that these are halves?"

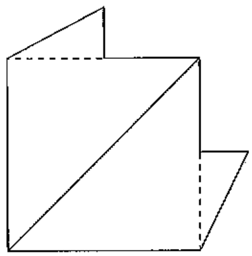


Neil said, "I know they are halves because they are symmetrical. I didn't actually fold it, but it's obvious that if I fold it on the line, the two sides would match exactly."

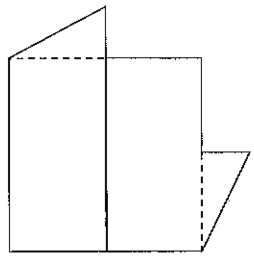
I remembered seeing Matthew with a different proof, so I asked him to share his. Matthew explained, "I divided it in the same place, but I have a different reason. I split the 'extra pieces' off each side to make a

*"Crazy Cakes for Two" from C. Tierney, M. Oganowski, A. Rubin, and S. J. Russell, *Different Shapes, Equal Pieces*, a grade 4 unit of *Investigations in Number, Data, and Space* (Glenview, IL: Scott Foresman, 1998).

square. I knew I could split a square into halves with this line [corner to corner]. Then I just have these two extra pieces that match. One goes with each half."



Matthew's split



Mackenzie's split

Mackenzie said, "I proved it kind of like Matthew, but I split it in a different place. I thought about the square, like Matthew did, and I split it like this [top to bottom]. Then I had the extra pieces on each half."

I summarized what we had so far. "We have two different ways of splitting the cake, and two different ways of proving one of those splits. But I see one way those are all the same: You are matching regions of one half with regions of the other half. Neil exactly matched one half with the other. They are the same size and shape and fit exactly on top of each other. Matthew matched the two halves of the square with each other, and then the two extra pieces. Mackenzie did not split her cake symmetrically, but she still matched regions. She matched the two halves of the square with each other, and then the two extra pieces. If you can match the areas of the two sides, you have halves."

We moved on to another shape, and I asked Marie to show where to cut it. Marie usually lacks confidence in her mathematical ability, often prefacing any comments with, "This is probably wrong, but...." She had obviously enjoyed the crazy cake activity, though, and I had overheard her explaining it clearly to her partner during the work period. She had been excited as she found and proved her solutions. This seemed like a good opportunity for her to share something she was sure of.

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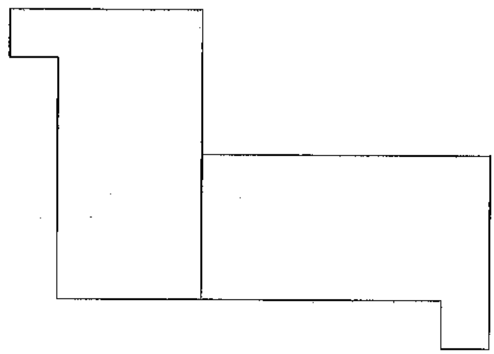
Marie drew a single line and told us, "When I did this before, I cut the shape out. When I cut on this line, I could put the two pieces together. They were just the same." Most students had the same solution, although only a few had actually cut the pieces.



Marie's split

We worked with several other cake shapes and, as it turned out, all the solutions presented were correct, although it wasn't always obvious when they were first presented. When students solved these problems for themselves, they found only one solution and then moved on to the next problem. In our discussion, when we saw a variety of solutions for the same cake, students actually said out loud, "I never would have thought of doing it that way." This also happened when we saw a variety of proofs for the same solution. I sensed that everyone was seeing something (not necessarily the same thing) for the first time. The solutions and the strategies for proving them held a degree of novelty that kept everyone engaged.